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Cavitation and the state of stress in a flowing liquid

By **DANIEL D. JOSEPH**

Department of Aerospace, Engineering and Mechanics, University of Minnesota, Minneapolis, MN 55455, USA

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The problem of the inception of cavitation is formulated in terms of a comparison of the breaking strength or cavitation threshold at each point in a liquid sample with the principal stresses there. A criterion of maximum tension is proposed which unifies the theory of cavitation, the theory of maximum tensile strength of liquid filaments and the theory of fracture of amorphous solids. Liquids at atmospheric pressure which cannot withstand tension will cavitate when and where tensile stresses due to motion exceed one atmosphere. A cavity will open in the direction of the maximum tensile stress which is 45° from the plane of shearing in pure shear of a Newtonian fluid. Experiments which support these ideas are discussed and some new experiments are proposed.

1. Introduction

In previous papers (Joseph 1995; Joseph, Huang & Candler 1996) I drew attention to the fact that the pressure in a flowing incompressible liquid is not a fundamental dynamic variable; at each point in the liquid the state of stress is determined by three principal stresses. In Newtonian fluids the pressure is the negative of the mean of these stresses (1.6); in non-Newtonian fluids the pressure is an unknown field variable whose relation to the principal stresses depends on the choice of a constitutive equation.

We may generally express the stress $\boldsymbol{T}$ by a constitutive equation of the form

$$\boldsymbol{T} = -p\mathbf{1} + \boldsymbol{\tau}[\boldsymbol{u}] \tag{1.1}$$

where the part $\boldsymbol{\tau}$ of $\boldsymbol{T}$ which is characterized by a constitutive equation can be regarded as functional of the velocity $\boldsymbol{u}$. For incompressible liquids, the conservation of mass is expressed by

$$\nabla \cdot \boldsymbol{u} = 0 \tag{1.2}$$

and the conservation of momentum by

$$\rho\left[\frac{\partial \boldsymbol{u}}{\partial \tau} + \boldsymbol{u} \cdot \nabla \boldsymbol{u}\right] = -\nabla p + \nabla \cdot \boldsymbol{\tau}[\boldsymbol{u}]. \tag{1.3}$$

Equations (1.2) and (1.3) are four equations for three components of velocity and the pressure p is an additional unknown which we need to close the system.

For Newtonian liquids

$$\boldsymbol{\tau}[\boldsymbol{u}] = 2\eta \boldsymbol{D}[\boldsymbol{u}], \tag{1.4}$$

where $\mathbf{D}[\mathbf{u}]$, the rate of strain, is the symmetric part of the gradient of velocity, η is the viscosity, and

$$\text{Trace}\,\mathbf{D}[u] = \nabla\cdot\mathbf{u} = 0. \tag{1.5}$$

As a consequence of (1.5),

$$p = -\tfrac{1}{3}\,\text{Trace}\,\mathbf{T}. \tag{1.6}$$

More generally, Trace $\boldsymbol{\tau} \neq 0$ and

$$p = -\tfrac{1}{3}\,\text{trace}\,(\mathbf{T} - \boldsymbol{\tau}) \tag{1.7}$$

depends on the constitutive equation, the choice of the functional relating $\boldsymbol{\tau}$ to $\mathbf{u}$.

Though it is true that a liquid at rest, in which all the stresses are all equal to $-p$, can make sense of (1.6), a moving liquid cannot average the principal stresses as is required by (1.6), and (1.7) is even more a consequence of how we choose to define $\boldsymbol{\tau}$ than a fundamental quantity which can be felt at a point by the liquid.

2. Cavitation index

The idea is that the state of stress at each point of a moving liquid is determined by the three principal stresses

$$T_{11} \geqslant T_{33} \geqslant T_{22} \tag{2.1}$$

and not by the pressure given by (1.7). Criteria for the inception of cavitation in liquids are here framed in terms of the principal stresses (2.1) rather than the pressure (1.7) used traditionally. Most of the traditional studies are framed in terms of a cavitation index. The utility of a cavitation index based on pressure is not evident. In one formulation, the index is given by

$$k = \frac{\tilde{p} - p_c}{\frac{1}{2}\rho U^2}, \tag{2.2}$$

where $\tilde{p}$ is the static pressure in the main stream, U is the bulk velocity of the fluid and $\tilde{p}$ and U are measured at the instant that cavitation commences.

Milne-Thompson (1960, chapter XII) considers the vapour cavity behind a moving cylinder and he forms an index which he attributes to Prandtl

$$k = \frac{\tilde{p} - p_c}{\frac{1}{2}\rho U^2} = \frac{V^2 - U^2}{U^2}, \tag{2.3}$$

where $\tilde{p}$ is the pressure at ∞, p_c is the pressure in the cavity and V is the fluid speed on the cavity wall. By Bernoulli's theorem

$$\tilde{p} + \tfrac{1}{2}\rho U^2 = p_c + \tfrac{1}{2}\rho V^2. \tag{2.4}$$

This index shows that a cavity will form on the top of the cylinder where the flow is fastest.

Some limitations of the cavitation index are widely appreciated by the cavitation community. The important discovery (Arakeri & Acosta 1973) has been that, even though viscous stresses are thought to have a negligible effect in cavitating water flows, viscosity has a major impact on flow structure, like separation points, which have an impact on the pressure distribution as a consequence of which cavitation is also affected. Franc & Michel (1985) found that in the flow of water over circular and elliptic cylinders cavities do not detach from the body at the minimum pressure point, but behind a laminar separation. They noted a direct link between separation of the laminar boundary layer and the initial signs of cavitation both of which are located in the recirculation zone downstream of the detachment.

Other limitations of the cavitation index, like cavitation induced by high flow-induced tensile stresses, have not been considered by the cavitation community. Such stresses, though typically small in water, could reach sensible values in more viscous liquids, and even in special flows of water. High tensile stresses on water threads stripped of a drop by high-speed air may also cause cavitation (Joseph, Huang & Candler 1996). The possibility of flow-induced tensile stresses due to stretching motions at a point of separation in cavitating flows at the inlet of holes in atomizers ought to be considered.

3. Principal stresses and cavitation

The state of stress rather than its average value is fundamental for all the motions of an incompressible fluid. Here, however, we focus on the inception of cavitation and not on the shape and motion of an open cavity. Even though criteria for cavitation ought to be based on the principal stresses and not the pressure, it is useful to introduce a pressure as the mean normal stress as in a Newtonian liquid and to define it that way for non-Newtonian liquids. If we write

$$\boldsymbol{T} = -\pi \mathbf{1} + \boldsymbol{S} = -p\mathbf{1} + \boldsymbol{\tau}, \tag{3.1}$$

where p is given by (1.7) and $\boldsymbol{S}$ is the stress deviator,

$$\pi = -\tfrac{1}{3}\ \mathrm{Trace}\ \boldsymbol{T}, \quad \mathrm{Trace}\ \boldsymbol{S} = S_{11} + S_{22} + S_{33} = 0. \tag{3.2}$$

Since $S_{11} \geqslant S_{33} \geqslant S_{22}$ we have

$$S_{11} > 0 \text{ and } S_{22} < 0, \tag{3.3}$$

where

$$S_{11} - S_{22} > 0 \tag{3.4}$$

is largest in the coordinate system in which $\boldsymbol{T}$ is diagonal.

Consider now the opening of a small cavity. It is hard to imagine very large differences in the pressure of the vapour in the cavity so that the cavity should open in the direction where the tension is greatest. The idea that vapour cavities open through tension is endemic in the cavitation community, but is seems not to have been noticed before that this idea requires one to consider the state of stress at a point and, at the very least, to determine the special principal axes coordinates in which the tension is maximum. To remind us of this important point we shall call $\zeta(\theta)$ the special coordinate system in which the orthogonal transformation $\boldsymbol{Q}$ diagonalizes $\boldsymbol{T}$ (and $\boldsymbol{S}$):

$$\boldsymbol{Q}^T \boldsymbol{T} \boldsymbol{Q} = \mathrm{diag}(T_{11}, T_{22}, T_{33}). \tag{3.5}$$

Here θ in $\zeta(\theta)$ stands for the direction cosines in the diagonalizing transformation, and θ is the diagonalizing angle for the two-dimensional rotation. The rotation of $\boldsymbol{T}$ is an important part of the theory of cavitation which has not been considered before.

In two dimensions the components of the stress deviator in $\zeta(0)$ are given by

$$[\boldsymbol{S}] = \begin{bmatrix} S_{11} & S_{12} \\ S_{12} & -S_{11} \end{bmatrix}. \tag{3.6}$$

The angle θ that diagonalizes $\boldsymbol{S}$ is given by

$$\sin 2\theta = S_{12}/(S_{12}^2 + S_{11}^2)^{1/2}, \quad \cos 2\theta = S_{11}/(S_{12}^2 + S_{11}^2)^{1/2} \tag{3.7}$$

and

$$[\boldsymbol{S}] = (S_{12}^2 + S_{11}^2)^{1/2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \tag{3.8}$$

The largest stress component in the principal value coordinate system is

$$T_{11} + \tfrac{1}{2}(T_{11} + T_{22}) = S_{11}; \tag{3.9}$$

the smallest component is

$$T_{22} + \tfrac{1}{2}(T_{11} + T_{22}) = -S_{11} \tag{3.10}$$

and

$$T_{11} - T_{22} = 2S_{11}. \tag{3.11}$$

We call T_{11} the maximum tension and T_{22} is the minimum tension. If the maximum tension is negative, it is compressive; the minimum tension is even more compressive.

If the cavitation (outgassing) threshold p_c is above $\pi - S_{11}$ but below π the cavity will appear when and where the tension due to motion is large enough; if this threshold is greater than $\pi - S_{22} (S_{22} < 0)$ then the cavity will open only at those points where no component of the total stress is larger than the cavitation threshold; this is the minimum tension criterion and in neither case is the criterion framed in terms of the pressure π alone.

If a cavitation bubble opens up, it will open in the direction of maximum tension. Since this tension is found in the particular coordinate system in which the stress is diagonal, the opening direction is in the direction of maximum extension, even if the motion is a pure shear. It may open initially as an ellipsoid before flow vorticity rotates the major axis of ellipsoid away from the principal tension axis of stress, or it may open abruptly into a 'slit' vacuum cavity perpendicular to the tension axis before vapour fills the cavity as in the experiments of Kuhl *et al.* (1994) (see figures 2 and 3).

These features in the two-dimensional problem have an immediate and obvious extension to three dimensions.

4. Cavitation criteria

In what follows I am going to assume that the breaking stress is a given parameter which can be defined at each point in a liquid; we then compare the state of stress in a moving liquid at the point with p_c to form a cavitation criterion.

The cavitation threshold used in the literature is framed in terms of a mean stress

$$\pi = -\tfrac{1}{3}(T_{11} + T_{22} + T_{33}). \tag{4.1}$$

Cavitation will occur when $\pi - p_c < 0$ and not when $\pi - p_c > 0$. The mean stress may be a good estimate for breaking thresholds, but it has no physical meaning in a moving fluid since the fluid cannot average its stresses.

Two cavitation thresholds based on the maximum tension T_{11} and minimum tension T_{22} in three dimensions can be considered; recall that the deviatoric stresses are such that

$$S_{11} > 0, \; S_{22} < 0 \tag{4.2}$$

so that $T_{22} = S_{22} - \pi$ is the minimum tension.

The *maximum tension* criterion is given by

$$B_{11} \overset{\text{def}}{=} T_{11} + p_c = S_{11} - \pi + p_c > 0. \tag{4.3}$$

In this case

$$\pi - p_c < S_{11} \tag{4.4}$$

and since the tension $S_{11} > 0, \pi - p_c$ could be larger than zero and the liquid would still cavitate. If (4.3) holds and

$$B_{22} \overset{\text{def}}{=} T_{22} + p_c < 0 \tag{4.5}$$

then relative to the threshold p_c, the stress B_{11} is in 'tension' and $B_{22} < 0$ is a 'compression'.

If (4.3) holds and

$$B_{22} > 0 \tag{4.6}$$

then all three of the relative principal stresses B_{11}, B_{22}, B_{33} are positive and a cavity will open. This is the *minimum tension criterion*. This criterion for cavitation is more severe than the classical one which requires that the average value of these relative stresses be positive.

The archival literature on cavitation allows only for breaking in tension, though the state of stress at a point which ought to be considered, has not been considered. Typically the discussion of cavitation is framed in the context of the breaking strength of liquids; the main conclusion is that liquids may withstand very large tensions if impurities and nucleation sites are suppressed. There is a vast literature on the tensile strength of liquids some of which may be found in the book by Knapp, Daily & Hammitt (1970) who say that '… Measurements have been made by several different methods and are too numerous to report completely', and in other books on cavitation.

Knapp *et al.* (1970) have considered whether the cavitation threshold ought to be framed in terms of the vapour pressure or the tensile strength of liquids, concluding for the latter. They say that

> … the elementary concept of inception is the formation of cavities at the instant the local pressure drops to the vapor pressure of the liquid. However, the problem is not so simple. Although experiments show inception to occur near the vapor pressure, there are deviations of various degrees with both water and other liquids that are not reconcilable with the vapor-pressure concept. We define the vapor pressure as the equilibrium pressure, at a specified temperature, of the liquid's vapor which is in contact with an *existing* free surface. If a cavity is to be created in a homogeneous liquid, the liquid must be ruptured, and the stress required to do this is not measured by the vapor pressure but is the *tensile strength* of the liquid at that temperature. The question naturally arises then as to the magnitudes of tensile strengths and the relation these have to experimental findings about inception.

A similar point of view was expressed by Plesset (1969)

> … A central problem in cavitation and boiling is how macroscopic vapor cavities can form when moderate tensions are applied to the liquid. The theory of the tensile strength of pure liquids predicts that a vapor cavity will form only when the liquid is under extremely large tensions; as an equivalent effect the theory also predicts that vapor bubbles appear in boiling only when the liquid has very large superheats. Since these large tensile strengths and superheats are not observed, the idea of nuclei has been introduced. These nuclei are in some sense holes in the liquid which are already beyond molecular dimensions and which may therefore grow into macroscopic bubbles under moderate liquid tensions.

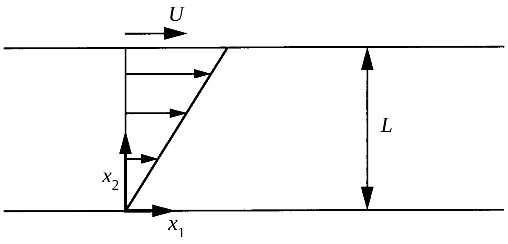

FIGURE 1. Plane Couette flow between walls.

Brennen (1995) notes that '... This ability of liquids to withstand tension is very similar to the more familiar property exhibited by solids and is a manifestation of the elasticity of a liquid'. Of course the elasticity of liquids, solid-like behaviour, could occur only in time so short that the configurations of molecules is not changed by flow, as could be expected in a cavitation event. Fisher (1948) notes that '... Glass and other undercooled liquids may fail by the nucleation and propagation of cracks, rather than of bubbles as do more mobile liquids'. Nucleation and propagation of cracks have been realized in the experiments of Kuhl *et al.* (1994) discussed in §8.

The theory of cavitation, the tensile strength of liquids and the fracture of amorphous solids may be framed in a unified manner in which the breaking strength of the material is defined in terms of tensile stresses along the principal axes of stress. Glass at different temperatures is a perfect material for these considerations. At high temperatures the molten glass flows and under the right conditions, flow bubbles ought to open at a weak spot in the direction of the principal tension. Low-temperature glass is an amorphous solid and we can imagine a crack to be initiated under tension at the same weak spot. Glass at intermediate temperatures may exhibit as yet unknown properties between cavity formation and crack propagation.

The nucleation of a cavity can occur as a sudden and not a continuous event. The fluid must first rupture; then it fills with vapour or gas and flows as in the experiments of Israelachvili and his collaborators described in §8. To open a cavity, the liquid must be supersaturated; practically this supersaturation can be achieved by lowering the pressure or by tensions created by flow. If the mean normal stress in a liquid is of the order of one atmosphere, the liquid will be put into tension when and where the tensile component of the flow-induced extra stress is larger than 10^5 Pa. Tap water might be expected to nucleate vapour or gas bubbles at points at which the flow-induced tensions exceed 1 atmosphere $\approx 10^5$ Pa. On the other hand, for flows with large capillary pressures or for fluids, which can withstand tension, larger flow-induced tensions, say 10^6 Pa, are required.

5. Cavitation in shear

Consider plane shear flow between parallel plates as in figure 1.

The stress in this flow is given by

$$\begin{bmatrix} T_{11} & T_{12} & 0 \\ T_{12} & T_{22} & 0 \\ 0 & 0 & T_{33} \end{bmatrix} = -\pi \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \eta \begin{bmatrix} 0 & U/L & 0 \\ U/L & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad (5.1)$$

where $\pi = \frac{1}{3}(T_{11} + T_{22} + T_{33})$ is determined by the 'pressurization' of the apparatus.

The angle which diagnonalizes $\boldsymbol{T}$ is given by (3.7) as $S'_{12} = 0$ or

$$\cos 2\theta = 0, \quad \theta = 45^\circ.$$

(In the experiments on the break-up of viscous drops in plane shear flow done by G. I. Taylor (1934), the drops first extend at 45° from the direction of shearing.)

Then, using (5.1), in principal coordinates, we have

$$\begin{bmatrix} T_{11}+\pi & 0 & 0 \\ & T_{22}+\pi & 0 \\ 0 & 0 & T_{33}+\pi \end{bmatrix} = \eta\frac{U}{L}\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{5.2}$$

and

$$B_{11} = p_c - \pi + \eta\frac{U}{L}, \tag{5.3}$$

$$B_{22} = p_c - \pi - \eta\frac{U}{L}. \tag{5.4}$$

The difference between the largest and smallest stresses is

$$B_{11} - B_{22} = 2\eta\frac{U}{L}. \tag{5.5}$$

This difference is of the order of one atmosphere of pressure if

$$2\eta\frac{U}{L} = 10^6\frac{\text{dynes}}{\text{cm}^2}. \tag{5.6}$$

If $\eta = 1000$ P, $U = 10$ cm s^{-1} and $L = 10^{-1}$ cm, we may achieve such a stress. It is possible to imagine such a shearing motion between concentric rotating cylinders filled with silicon oil, though the conditions are severe. If we could depressurize the system so that a threshold of pressure less than one atmosphere were required, we might see cavities appear in shear flow when $B_{11} > 0$ and $B_{22} < 0$. I am not aware of reports of cavities forming in shear flows, but the conditions required are at the border of realistic experiments and may have escaped detection. Experiments of this kind ought to be tried.

Cavities formed in shear flows have been reported recently in a paper by Archer, Ternet & Larson (1997). They note that '... the shear stress catastrophically collapses if the shear rate is raised above a value corresponding to a critical initial shear stress of around 0.1–0.3 MPa. ... in polystyrene, bubbles open up within the sample; as occurs in cavitation. Some similarities are pointed out between these phenomena and that of 'lubrication failure' reported in the tribology literature.' The critical stress 0.1–0.3 MPa = 1–3 atmospheres is just what might have been guessed for cavitation under shear.

6. Cavitation in extension

We have argued that cavities always appear in the extensional flows defined in principal axes coordinates even when the flow is pure shear. However, the direct creation of a pulling flow without rotation (vorticity) may lead to a higher level of dynamic stresses than could be otherwise achieved. Let us suppose that a small-diameter thread open to the atmosphere is anchored at a solid wall at $x = 0$ and is being pulled out at a constant rapid rate $\overset{\circ}{S}$ in the direction x:

$$u = \overset{\circ}{S}\, x, \quad v = -\tfrac{1}{2}\,\overset{\circ}{S}\, y, \quad \omega = -\tfrac{1}{2}\,\overset{\circ}{S}\, z. \tag{6.1}$$

The thread is in tension when $\overset{\circ}{S}$ is large enough:

$$T_{11} = -\pi + 2\eta \frac{\partial u}{\partial x} \approx -p_a + 2\eta\, \overset{\circ}{S}, \tag{6.2}$$

where, for very thin threads $\pi \approx -p_a + \gamma/R$ where p_a is atmospheric pressure, γ is surface tension and R is the radius. According to the maximum tension criterion (4.1) cavities will form in the thread, and the thread may actually break, when

$$B_{11} \approx p_a + \frac{\gamma}{R} - p_c - 2\eta\, \overset{\circ}{S} < 0. \tag{6.3}$$

If we neglect surface tension the stretch rate $\overset{\circ}{S}$ for breaking can be estimated, assuming that the thread cannot sustain a tension, by $p_c = 0$; then

$$\overset{\circ}{S} > 10^6/2\eta (\mathrm{s}^{-1}).$$

For very viscous threads, say $\eta = 500$ P, the stretch rate for breaking

$$\overset{\circ}{S} > 10^5 (\mathrm{s}^{-1})$$

is rather large.

The extensional flow (5.5) with a time-dependent $\overset{\circ}{S}$ may be used to model the motion emanating from a stagnation point at the centre of the neck in a collapsing capillary filament. Lundgren & Joseph (1997) found that the neck is of parabolic shape and its radius collapses to zero in a finite time. During the collapse the tensile stress due to viscosity increases in value until at a certain finite radius which is about 1.5 μm for water in air, the stress in the throat passes into tension, presumably inducing cavitation there. On the other hand, Eggers' (1993), Brenner *et al.* (1997) model of capillary breakup does not seem to give rise to tension at the throat and though experiments can be done, cavitation has not been considered.

7. Breaking tension of polymer strands

Another example of breaking of viscous threads in tension has been documented in experiments by Wagner, Schulze & Göttfert (1996) on the drawability of polymer melts.

In these experiments the tensile force needed to elongate an extruded polymer melt is measured as a function of the draw ratio $V = v/v_0$, where v_0 is the velocity of the spinline at the die and v is the velocity of the spinline at the takeup wheels. The tensile force F is measured at the wheel and the stress in the strand at the wheel is said to be given by

$$\sigma = FV/A_0,$$

where A_0 is the area of the cross-section of the die hole. V and F increase together and at a certain critical F_B (and V_B) the strand breaks. The remarkable feature of this breaking is that the breaking stress σ_B is independent of the extrusion pressure (the wall shear stress) and temperature. Wagner *et al.* (1996) conclude that the breaking stress σ_B is a 'pure material constant'.

The breaking stress in their LDPE sample A18 ($\eta_0 = 10^4$ Pa s) is

$$\sigma_B \approx 10^6 \text{ Pa}.$$

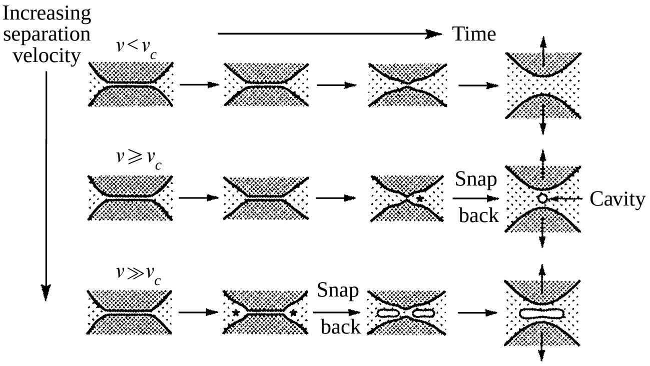

FIGURE 2. Schematic illustration of the separation of two observed mica surfaces at progressively increasing separation velocities as ascertained from the FECO fringe pattern and direct optical microscope visualization. The most likely places where recoil and damage occurred are shown by the starred points (*). Top row: $v < v_c$: smooth separation; no cavities. Middle row: $v \geqslant v_c$: abrupt separation; cavity and damage form at center. Bottom row: $v \gg v_c$: abrupt separation; cavities and damage form at rim (crater-like).

The breaking stress in the HDPE sample H50 ($\eta_0 > 4.8$ Pa s) is

$$\sigma_B \approx 1.1 \times 10^6 \text{ Pa}.$$

Atmospheric pressure is roughly

$$p_a \approx 1.1 \times 10^5 \text{ Pa};$$

the pressure in the thread is somewhat larger than this because of surface tension. The radius of the die is 1 mm; if the thread thins by a factor 10 or more the surface tension addition to pressure in the thread will be sensible. It is nevertheless certain that the strand is in tension when it breaks.

8. Cavitation experiments at the nanoscopic level

Chen & Israelachvili (1991) and Kuhl *et al.* (1994) have done important direct visualization studies of cavitation of ultrathin nanometer liquid films using the surface forces apparatus technique. They are able to visualize cavitation between mica surfaces in approach-separation and shearing motions. They noticed that vapour cavities developed when two curved surfaces are moved away from each other faster than some critical velocity v_c. In the experiments described by Kuhl *et al.* (1994), the liquid between 1 cm radius hemispheres of mica was a low molecular weight, Newtonian, 180 P polybutadiene and the separating motions can be thought to give rise to extensional motions like those described in (5.5).

Chen & Israelachvili say that

> We have found that cavitation bubbles can occur either totally within the liquid, that is, away from the surfaces, or at the solid-liquid interfaces. The adhesion of untreated (polar) mica surfaces to the PBD liquid is stronger than the cohesion between the liquid molecules themselves (“wetting” conditions); hence, the cavities form totally within the liquid. In contrast, for surfaces coated with a surfactant monolayer, the nonpolar solid-liquid adhesion is weaker … and the cavities form at the interfaces.

A qualitative description of their observation for the case of strong adhesion is described in the caption for figure 2.

The experiments of Israelachvili and his associates show that cavities open in tension at a threshold value of the extensional stress and that the formation of cavities is analogous to the fracture of solids, with the added caveat that the liquid can flow into the crack immediately after fracture. In the words of Kuhl *et al.* (1994)

> If the speed of separation is increased, the surfaces become increasingly more pointed just before they rapidly move apart. Then, above some critical speed v_c (here about 100 μm/s) a completely new separation mechanism takes over, as shown in Figure 3 [Reproduced here as figure 3.]. Instead of separating smoothly, the liquid 'fractures' or 'cracks' open like a solid. It is known that when subjected to very high shear rates, liquids begin to behave mechanically like solids, for example, fracturing like a brittle solid. In our experiments, the point and time at which this 'fracture' occurred was just as the surfaces were about to separate from their most highly pointed configuration (Fig. 3C) - for had the separation velocity been any smaller than v_c they would have separated smoothly without fracturing. We consider that in the present case, the 'fracturing' or 'cracking' of the liquid between the surfaces must be considered synonymous with the "nucleation" or "inception" of a vapor cavity.

The stretch rate may be underestimated by v_c/l where $2l$ is the shortest distance between the mica surfaces. To get cavitation it is necessary to cross a stress threshold which is consistent with the observation that '... The thicker the initial film thickness the higher the value of v_c...'

Of course, the analysis of steady extension in §6 does not apply to the highly unsteady cavitation being described here. An estimate of the stress level at cavitation can nevertheless be composed as

$$2\eta\, \overset{\circ}{S}$$

with $\overset{\circ}{S}(t)$ the maximum value of the stress rate between $t = 10.00$ s when there is no cavity and $t = 10.01$ s when a cavity has definitely opened. It may be optimistic, but certainly possible, that the between distance the bump on the top mica surface and the bottom surface changes by 1 nm in 10^{-4} to 10^{-5} s. Then, with $2\eta = 36$ Pa s we get

$$3.6 \times 10^5 < 2\eta\, \overset{\circ}{S} < 3.6 \times 10^6 \text{ Pa}$$

which is greater than atmospheric pressure. A tension of this magnitude could open up a vacuum cavity. According to Kuhl *et al.* (1994) '... When a cavity initially forms and grows explosively, it is essentially a vacuum cavity since dissolved solute molecules or gases have not had time to enter into the rapidly growing cavity'. The final collapse of the cavity is slower because the cavity fills with vapour.

9. Conclusions

A summary of the main points in this paper is listed below.

- The pressure in incompressible Newtonian fluids is the mean normal stress. The stress is decomposed into a pressure and stress deviatior with a zero trace. The pressure in incompressible non-Newtonian liquids is given by the constitutive equation and has no intrinsic significance. Cavitation criteria for liquids in motion must be based on the stress and not on the pressure. The liquid cannot average its stresses or recognize the non-unique quantity called pressure in non-Newtonian fluids.
- It is convenient for the study of cavitation of flowing liquids to decompose the stress into a deviator and mean normal stress. The deviator has positive and negative

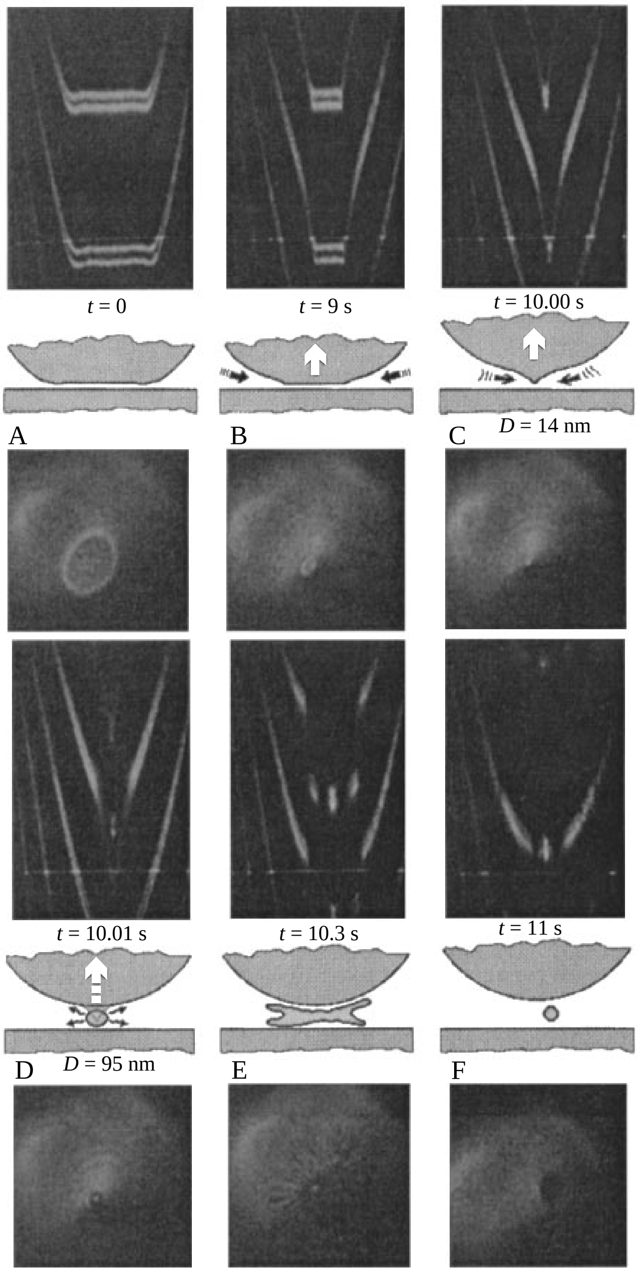

FIGURE 3. Surfaces separating at high speed, $v > v_c$, showing FECO fringes (top), schematic side-view (middle), and optical microscope view (bottom) of the surfaces. Homogeneous nucleation of a vapour cavity is shown in D ($t = 10.01$ s), after which the cavity grows rapidly and then collapses (D to F). Note that in picture F (1 s after inception) the cavity has still not totally disappeared (evaporated or collapsed). Reproduced from Kuhl *et al.* (1994).

normal stresses, deviating from the average. The most positive value of principal stresses is the maximum tension. The stress in non-Newtonian liquids should also decompose the stress into average and deviator.

- A cavitation bubble will open in the direction of maximum tension in principal coordinates. The angles defining the principal axis determine how a cavity will open; angles are important.
- A liquid can cavitate in shear. However, it is pulled open by tension in the direction defined by principal stresses; Newtonian liquids in pure plane shear will open 45° from that direction.
- Cavitation in a flowing liquid will occur at a nucleation site when the maximum tensile stress in principal axes coordinates is smaller than the cavitation pressure.
- Cavitation can be a fast, non-equilibrium event resembling fracture in which the cavity first opens and then fills with vapour and/or gas.
- Outgassing is cavitation of liquid gas in solution.

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REFERENCES

Arakeri, V. H. & Acosta, A. 1973 Viscous effects on the inception of cavitation on axisymmetric bodies. *Trans. ASME* I: *J. Fluids Engng* **95**, 519–528.

Archer, L. A., Ternet, D. & Larson, R. G. 1997 "Fracture" phenomena in shearing flow of viscous liquids. *Rheol Acta* **36**, 579–584.

Brennen, C. E. 1995 *Cavitation and Bubble Dynamics*. Oxford University Press.

Brenner, M. P., Eggers, J., Joseph, K., Nagel, R. & Shi, X. D. 1997 Breakdown of scaling in droplet fission at high Reynolds numbers. *Phys Fluids* **9**, 1573–1590.

Chen, Y. & Israelachvili, J. 19XX New mechanism of cavitation damage. *Science* **252**, 1157-1160.

Eggers, J. 1993 Universal pinching of 3D axisymmetric free-surface flow. *Phys. Rev.* E **71**, 3458–3460.

Fisher, J. C. 1948 The fracture of liquids. *J. Appl. Phys.* **19**, 1062–1067.

Franc, J. P. & Michel, J. M. 1985 Attached cavitation and the boundary layer; experimental investigation and numerical treatment. *J. Fluid Mech.* **154**, 63–90.

Hoyt, J. W. & Taylor, J. J. 1981 A photographic study of cavitation in jet flow. *Trans. ASME* **103**, 14–18.

Joseph, D. D. 1995 Cavitation in a flowing liquid. *Phy. Rev.* E **51**, 1649–1650.

Joseph, D. D., Huang, A. & Candler, G. 1996 Vaporization of a liquid drop suddenly exposed to a high speed air stream. *J. Fluid Mech.* **310**, 223–236.

Knapp, R., Daily, J. W. & Hammit, F. 1970 *Cavitation*. McGraw Hill.

Kuhl, T., Ruths, M., Chen, Y. & Israelachvili, J. 1994 Direct visualization of cavitation and damage in ultrathin liquid films. *J. Heart Valve Disease* **3**, (suppl. I) 117–127.

Lundgren, T. S. & Joseph, D. D. 1997 Capillary collapse & rupture. Submitted to *Phys. Rev. Lett.* (see http://www.aem.umn.edu/people/faculty/joseph/papers/downcap.html).

Milne-Thompson, L. M. 1960 *Theoretical Hydrodynamics*. Macmillan.

Plesset, M. 1969 Tensile strength of liquids. *Office of Naval Res. Rep.* 85–4.

Taylor, G. I. 1934 The formation of emulsions in definable fields of flow. *Proc. R. Soc. Lond.* A **146**, 501–523.

Wagner, M. H., Schulze, V. & Göttfert, A. 1996 Rheotens – Mastercurves and drawability of polymer melts. *Polymer Engng Sci.* **36**(7), 925–935.